Proceedings Letters

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General Energy Relations for Josephson Junctions

Abstract—The Josephson junction acts as a nonlinear lossless inductor and can be applied for mixing and parametric amplification in the microwave region. Since an alternating current can be generated in the junction by an applied dc voltage, a dc power flow must be taken into consideration. Energy relations for the Josephson junction are derived which are similar to the Manley–Rowe equations but with an additional term for the dc power. These equations show the possibility of realizing dc-pumped parametric amplifiers.

The Josephson junction can be used as generator, mixer, and detector in the microwave region [1]–[4]. Parametric amplification with Josephson junctions has been observed by Zimmer [5]. In this letter it will be shown that for the ideal Josephson junction described by the equations

$$i(t) = I_{\max} \sin \phi(t) \tag{1}$$

$$\dot{\phi}(t) = \frac{2ev(t)}{\hbar} \tag{2}$$

energy relations are valid, similar to those derived by Manley and Rowe [6], but with an additional term for the dc power.

Since the Josephson current is a single-valued function of the integral of the voltage over time, the Josephson junction is lossless and has the behavior of a nonlinear inductor. The energy stored in the junction is

$$w(\phi) = \frac{\hbar}{2e} I_{\max}(1 - \cos \phi). \tag{3}$$

Since the current and the energy are periodic functions of ϕ, a dc voltage can be applied to the junction. A dc voltage V_0 generates an alternating current with the angular frequency

$$\omega_0 = \frac{2e}{\hbar} V_0. \tag{4}$$

If a voltage with a dc component V_0 and with ac components at the angular frequencies $\omega_1, \cdots, \omega_\nu$ is applied, the current will have frequency components at $\omega_0 + n_1\omega_1 + \cdots + n_\nu\omega_\nu$, where the $n_1, \cdots, n_\nu$ are integers. If ω_0 is a combination frequency of the $\omega_1 \cdots \omega_\nu$, the current can have a dc component. In this case a dc power flow must be taken into consideration.

In the following we shall restrict ourselves to voltages of the form

$$v(t) = V_0 + \sum_{m=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty}{}' V_{mn} \exp j(m\omega_1 + n\omega_2)t$$

$$V_{-m-n} = V_{mn}^* \tag{5}$$

where ω_1 and ω_2 are incommensurable. The apostrophe denotes that the term with $m = n = 0$ shall be excluded from the summation. To get a power exchange between ac and dc, we impose the condition

$$\omega_0 = k\omega_1 + l\omega_2. \tag{6}$$

Introducing the independent variables

$$x = \omega_1 t, \qquad y = \omega_2 t \tag{7}$$

we get from (2), (4), (5), and (6)

$$\phi(x, y) = \phi_0 + kx + ly + \sum_{m=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty}{}' a_{mn} \exp j(mx + ny) \tag{8}$$

with

$$a_{mn} = -j \frac{(k\omega_1 + l\omega_2)V_{mn}}{(m\omega_1 + n\omega_2)V_0}. \tag{9}$$

The same frequencies occur in the frequency spectrum of the current as in the spectrum of the voltage if (6) is valid, and we can write

$$i(x, y) = I_0 + \sum_{m=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty}{}' I_{mn} \exp j(mx + ny). \tag{10}$$

The amplitudes I_{mn} are given by

$$I_{mn} = \frac{1}{4\pi^2} \int_0^{2\pi} \int_0^{2\pi} i(x, y) \exp[-j(mx + ny)] dx dy. \tag{11}$$

Multiplying (11) with $-jma_{mn}^*$, summing over m and n, and interchanging the order of summation and integration, we get

$$-j \sum_{m=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty}{}' mI_{mn}a_{mn}^* = -\frac{j}{4\pi^2} \int_0^{2\pi} \int_0^{2\pi} i(x, y) \sum_{m=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty}{}' ma_{mn}^* \cdot \exp[-j(mx + ny)] dx dy. \tag{12}$$

Introducing the partial derivative with respect to x of the complex conjugate of (8), we get

$$\begin{aligned} -j \sum_{m=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty}{}' mI_{mn}a_{mn}^* &= \frac{1}{4\pi^2} \int_0^{2\pi} \int_0^{2\pi} i(x, y) \frac{\partial}{\partial x}(\phi - \phi_0 - kx - ly) dx dy \\ &= \frac{1}{4\pi^2} \int_0^{2\pi} dy \int_{\phi(0,y)}^{\phi(2\pi,y)} i(\phi) d\phi - \frac{1}{4\pi^2} k \int_0^{2\pi} \int_0^{2\pi} i(x, y) dx dy. \end{aligned} \tag{13}$$

Since the integral of $i(\phi)$ over ϕ is periodic with 2π and $\phi(2\pi, y) - \phi(0, y) = 2k\pi$, the first integral on the right side of (13) vanishes. Introducing (11) into the second integral and the complex conjugate of (9) into the left side of (13), we get

$$\sum_{m=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} \frac{mI_{mn}V_{mn}^*}{m\omega_1 + n\omega_2} = -\frac{kI_0V_0}{k\omega_1 + l\omega_2}. \tag{14}$$

The dc power flowing toward the Josephson junction is

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$$P_0 = I_0 V_0 \tag{15}$$

and the active powers flowing toward the junction at the angular frequencies $m\omega_1 + n\omega_2$ are

$$P_{mn} = I_{mn} V_{mn}^* + I_{-m-n} V_{-m-n}^*. \tag{16}$$

Summing up the terms with positive and negative m in (14), we obtain the relation

$$\sum_{m=1}^{+\infty} \sum_{n=-\infty}^{+\infty} \frac{mP_{mn}}{m\omega_1 + n\omega_2} = -\frac{kP_0}{k\omega_1 + l\omega_2}, \tag{17}$$

and, in the same way, a second equation

$$\sum_{m=-\infty}^{+\infty} \sum_{n=1}^{+\infty} \frac{nP_{mn}}{m\omega_1 + n\omega_2} = -\frac{kP_0}{k\omega_1 + l\omega_2}. \tag{18}$$

Equations (17) and (18) show the possibility of realizing dc-pumped parametric amplifiers. Instead of pumping at a frequency ω_0, a dc voltage V_0 is applied to the junction that generates the frequency ω_0. The gain of a negative resistance parametric amplifier with a Josephson junction has been calculated [7]. Difficulties arise from the low impedance of Josephson junctions. For this reason it would be necessary to have Josephson junctions with low maximum currents. Since the impedance increases with the frequency, the application of Josephson junctions could be of some interest in the submillimeter region.

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Techniques for Minimizing MOSFET Gate Leakage Current

Abstract—Measurements made on the gate current of one side of a dual MOSFET show that special packaging and circuit techniques result in a substantial reduction in gate leakage by almost entirely eliminating header leakages. Gate currents as low as 20×10^{-18} A have been measured during the investigation. Details of the measuring techniques are also given.

INTRODUCTION

Input current measurements were made on ten samples of a specially packaged dual MOSFET (type MBH 1) consisting of two matched devices mounted in a TO-5 header as detailed in Fig. 1. The special features of the MBH 1 allow it to be employed in the circuit configuration shown in Fig. 2, which is a unity gain dc amplifier[1] whose offset voltage can be adjusted to zero. As the output voltage of the stage is connected to the header, no potential exists between the input gate and the header; thus the extrinsic component of the input current that normally flows through leakage paths provided by the header is virtually eliminated. The results of these measurements are given in Table I.

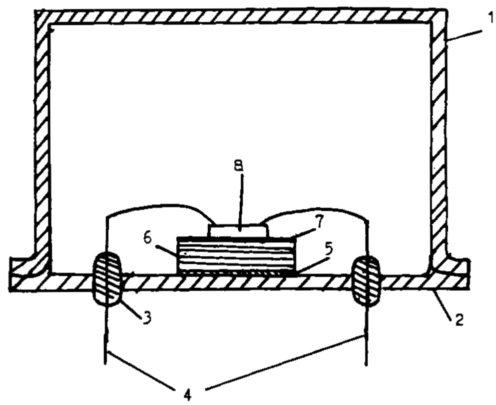

Fig. 1. MBH 1 construction. 1, metal casing; 2, metal base; 3, glass bead; 4, electrode lead (only two shown); 5, adhesive material; 6, ceramic insulator; 7, metalized layer; 8, MOSFET chip.N.B. Can brought out as a separate connection.

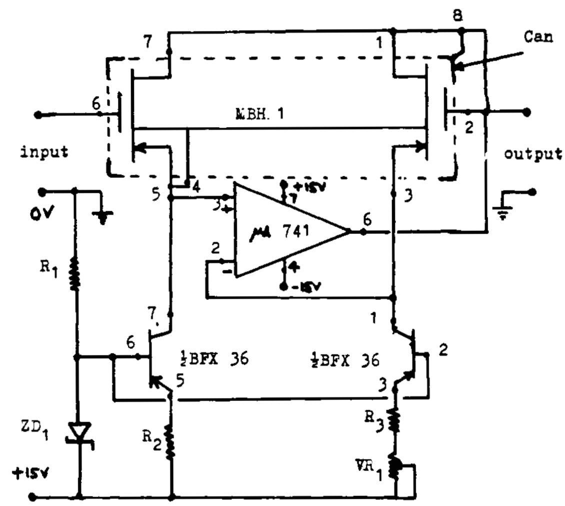

Fig. 2. Test circuit.

To assess the degree of improvement in the input current afforded by the MBH 1 construction, ten MBH 2 devices (conventionally packaged versions of the MBH 1 employing a header with a solid glass base) were also tested for input current. The results for these measurements are given in Table II.

MEASURING TECHNIQUES

For the lower range of gate currents, below 10^{-15} A, the gate current was deduced from observations made on the variation of the output voltage V_o of the amplifier with time. As the offset voltage is zero, the output voltage closely follows input voltage changes. To make a measurement, the input gate was open circuited allowing the gate current I_g to charge the input capacitance C_i of the amplifier. As no leakage paths to earth are present, the gate current is given by

$$I_g = C_i dV_o/dt. \tag{1}$$

Full details of this technique were reported previously,[2] together with a method for determining C_i.

For the higher range of currents, the amplifier containing the device under test was incorporated in the measuring circuit shown in Fig. 3, where A is a high-gain amplifier and R is a high-value resistor ($10^{12}\ \Omega$ in this case). Neglecting second-order quantities, the output voltage V_2 of this arrange-

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